Typical Moire Results and Analysis

The main objective of this portion of the paper is to illustrate the varying stress/strain fields in the vicinity of the hole arrays. Three configurations are considered. The first has two holes in a row parallel to the direction of load with both holes loaded equally. However, in less ideal conditions, some load misalignment exists, thus producing a nonuniform distribution of the load between the two pins. In order to simulate a worst condition, the holes were tested with individual loading. In the second case, two holes are in a row perpendicular to the direction of load and both holes are loaded equally. The third configuration considered has three holes in a staggered or triangular pattern. All of the holes were loaded equally and simultaneously. See Figs. 1-3 for typical results.

Summary and Conclusions

Table 1 compares the stress concentration factors for two of the hole arrays and load configurations studied.

Using a high-sensitivity interferometric moire technique, the stress/strain distribution was obtained for three different directions. The SCF was calculated at only one point located at the edge of the hole in the horizontal diameter for the two- and three-hole arrays. For the two-hole arrays in tandem, when both holes were loaded, the SCF for the upper hole was 2.9 times higher than that at the lower hole, indicating that the load sharing between both holes is not exactly given by a ratio of 2. In the case of two holes in tandem (lower hole loaded), the SCF for the lower hole was 1.84 times higher than the SCF for the upper hole. Some interaction between the two holes, one of them being loaded and the other acting as a stress raiser, defines the stress/strain field in the region between them, but the influence of the hole acting as a stress raiser on the loaded one is not very large. When the upper hole is loaded, the SCF for the upper hole produces a large influence in the stress/strain field of the lower hole. The ratio of SCF between both holes is 12. For the case of the two-hole array in parallel, there is a further reduction of the net section area, and an increase in the SCF was expected at the edge of the holes. In this case, the results showed a fairly uniform distribution of stress/strain around the hole.

In the case of the three-hole array in a staggered configuration, some asymmetry was observed for the SCF. For the upper hole, $k=2.32$ and 2.17 for the right and left holes, respectively, and for the inner edges of the two lower holes, $k=2.95$ and 1.54 for the right and left holes, respectively. This was an indication that the right half of the specimen had a higher SCF. Also, some high strain values were observed above the two lower holes in a symmetric position with respect to the specimen. The bearing area of the upper hole did have some effect on the strain field of the three holes.

If these results are compared with the results obtained by Herrera-Franco[6] and Cloud et al.[1] for a single pin-loaded hole, where the SCF at the edge of the hole was equal to $k=14$, then the advantage of utilizing a multihole array in a connection is evident by the reduction of SCF at each hole.

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Use of Curvilinear Fiber Format in Composite Structure Design

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Introduction

COMPOSITE structures are currently being designed to make use of multiple layers of fiber-reinforced material. The fibers in each layer are aligned straight and parallel with each other. Within a particular layer, the fiber orientation is fixed. This is opposed to having a fiber orientation that varies from point to point within a layer or group of layers. The idea of using fibers in anything other than a straightline format has been hampered by the inability to implement such a design. However, contemporary fiber handling techniques, such as multiaxis tow placement devices[1] may eliminate this barrier. As such, it is worthwhile to consider the gains, or losses, that accompany the use of curvilinear fiber formats. It would seem from the outset that by allowing the reinforcing medium the additional freedom to have its alignment vary from point to point depending on geometry and load path, more efficient structures can be designed. Though this may appear to be the case, each particular application of such an idea has to be studied. For example, a plate with a centrally located circular hole would be a prime candidate for utilizing the curvilinear format. For such a problem, the basic issues are the following: Can the tensile capacity and/or the buckling capacities of a plate with a central hole be improved by using the curvilinear format? How should the fiber orientation vary from point to point? What mechanisms are responsible for this improvement? How sensitive are the gains to geometry, e.g., plate-width-to-hole-diameter ratio? There are other questions that must be asked. This Note begins to discuss the notion of a curvilinear fiber format for the specific problem of a plate with a hole. It should be noted that some work has been done in the area of curvilinear reinforcement.[2,3]

Problem Description, Design Philosophy, and Method of Analysis

Consider a plate of length L, width W, with a central hole of diameter D, and loaded on opposite ends by uniformly distributed forces. Given that graphite-reinforced material with a fiber-volume fraction of 65% is available, how can the material be used most effectively to construct a plate with a given geometry and loading? Here, the discussion will focus on a tensile loading. Buckling of the improved tensile designs will be evaluated, but design with the curvilinear format for improved buckling is not discussed. For purposes of limiting the problem, the plates discussed will be restricted to 16 layers.

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The basic philosophy used to design the plate for tensile loading is to assume that some or all of the fibers should, in some sense, be aligned with the principal stress directions in the plate. Strictly speaking, principal stress directions are meaningless when discussing fiber-reinforced materials. More meaningful are the principal material directions. However, here, principal material directions, in conjunction with principal stress directions, will be utilized. Specifically, using an iteration technique, in certain layers the principal material directions and principal stress directions of those layers will be aligned. With this accomplished, two failure criteria are then used to evaluate the effectiveness of the tensile designs. The noninteracting maximum strain criterion, and the interacting stress-based Tsai-Wu criterion are used to study allowable load levels. Each criterion is applied on a layer-by-layer basis to predict failure load and failure mode. Two diverse criteria are used to determine if predicted behavior or performance gains are a function of the criterion used. For buckling, the lowest buckling load as computed from a linear buckling analysis is considered to be the compressive load capacity. Since the plates studied are quite thin, this load is much lower than any material compressive failure loads. Finite-element analyses are used throughout. Because of the lack of overall A_{16} and A_{26} terms, a quarter-plate analysis is used to effect the curvilinear design for improved tensile capacity. The specific finite-element code used was developed previously.[4] For buckling calculations, the commercially available code EAL[5] (Engineering Analysis Language) was used for the analysis of the full plate.[6]

Determination of Fiber Angles

It must be kept in mind that principal stress directions depend on material properties. What may be thought of as principal stress directions for an isotropic material, such as aluminum, will change when fibers are introduced into the material. In addition, the idea of principal stress directions for a laminate must be applied carefully. Each layer has principal stress directions, and these directions vary from layer to layer. Here, an iterative approach is used to find the fiber directions within a group of layers such that the fibers in those layers are everywhere aligned with the principal stress directions of those layers. The first iteration assumes that the fibers are everywhere aligned with the principal stress directions for an isotropic plate. The elastic properties used, of course, represent the nonisotropic composite. The stresses in each element are recomputed and another set of element principal stress directions determined. The fibers are aligned with these new directions and the stress analysis repeated. A third set of element principal stress directions is computed and the fibers realigned.

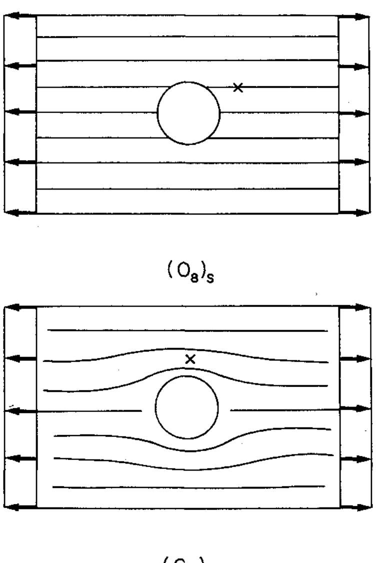

Fig. 1 Failure locations (x) for $(0_8)_S$ and $(C_8)_S$ plates.

This process of stress calculation and fiber realignment is repeated until the principal stress directions and the fiber directions are aligned to within a given tolerance. Specifically, the solution is considered converged when the quantity

$$(\theta - \phi)/\theta \tag{1}$$

<0.01 for each element. In Eq. (1), θ denotes the principal stress direction and ϕ denotes the fiber orientation, both measured relative to the loading direction. In practice, convergence was realized in 4 or 5 interactions. For the laminates studied here, the result of the iteration process was a principal stress direction for each element that was not significantly different than the direction from the original isotropic analysis.

Material Properties Used

The material properties used throughout the study are chosen to represent AS4/3501, an organic matrix graphite composite.[7] The elastic properties used are

$$E_1 = 138 \text{ GPa (20.0 Msi)} \qquad E_2 = 8.96 \text{ GPa (1.30 Msi)}$$

$$G_{12} = 7.10 \text{ GPa (1.03 Msi)} \qquad \nu_{12} = 0.30$$

$$\text{layer thickness} = 0.1 \text{ mm (0.005 in.)} \tag{2}$$

The failure strains used are

$$\epsilon_1^C = 10{,}500 \times 10^{-6}, \qquad \epsilon_1^T = 10{,}500 \times 10^{-6}$$

$$\epsilon_2^C = 23{,}000 \times 10^{-6}, \qquad \epsilon_2^T = 5{,}800 \times 10^{-6}$$

$$\gamma_{12}^S = 13{,}100 \times 10^{-6} \tag{3}$$

The failure stresses used are

$$\sigma_1^C = 210 \text{ Ksi}, \qquad \sigma_1^T = 210 \text{ Ksi}$$

$$\sigma_2^C = 30 \text{ Ksi}, \qquad \sigma_2^T = 7.5 \text{ Ksi}$$

$$\tau_{12}^S = 13.5 \text{ Ksi} \tag{4}$$

Results

The discussion to follow is for square plates with a plate-width-to-hole-diameter ratio of 3. The results presented do not depend strongly on geometry. To provide a basis for comparison, the results obtained are normalized by the results obtained for a quasi-isotropic $(\pm 45/0/90)_{2S}$ plate with the identical geometry. Since the quasi-isotropic configuration represents an acceptable conventional design, it is used as a datum.

Curvilinear Design for Tensile Loads

The first curvilinear design considered aligns all 16 layers with the principal stress direction at each point: an all-curvilinear design. This laminate is denoted $(C_8)_S$, the C standing for curvilinear. The laminate may be considered somewhat academic but much can be learned from it, and it is a good starting point. The $(C_8)_S$ laminate is the curvilinear counterpart to a $(0_8)_S$ laminate. A $(0_8)_S$ laminate fails at a load of 0.59, relative to the quasi-isotropic case, due to a shear failure of the matrix at the location shown in Fig. 1. The shear stress is generated because the uniform stress state at the ends of the plate must redistribute to a nonuniform state in the region of the hole. This realignment generates shear stresses. Aligning the fibers with the principal stress directions should eliminate that problem. The iteration process, to determine the fiber angles in each element, and subsequent analysis of the $(C_8)_S$ laminate confirms that this is the case. The failure analysis predicts that the material will fail in tension perpendicular to

Table 1 Tensile and buckling failure loads

Design	Failure load[a]	Failure mode	Buckling load[c]
$(\pm 45/0/90)_{2S}$	1.00 (0.99)	Fiber failure in 0-deg layer at net-section hole edge	1.00
$(0_8)_S$	0.59 (0.51)	Shear failure in matrix near hole but away from net-section (see Fig. 1)	0.46
$(C_8)_S$	1.01 (0.93)	Tension failure perpendicular to the fibers at net-section but away from hole edge (see Fig. 1)	0.52
$(O/C_7)_S$	1.89 (1.78)	Fiber failure in C layers at net-section hole edge	0.55
$(\pm 45/C_6)_S$	1.60 (1.66)	Fiber failure in C layers at net-section hole edge	0.87
$(\pm 45/0_6)_S$	1.27 (0.94)	Fiber failure in 0-deg layers at net-section hole edge[b]	0.84
$(\pm 45/C_2)_{2S}$	1.33 (1.38)	Fiber failure in C layers at net-section hole edge	0.93
$(\pm 45/0_2)_{2S}$	1.20 (1.22)	Fiber failure in 0-deg layers at net-section hole edge[b]	0.90

[a]Normalized by maximum strain criterion failure load for a quasi-isotropic laminate, number in parenthesis is Tsai-Wu prediction.
[b]Tsai-Wu criterion predicts some shear interaction.
[c]Normalized by the buckling load for a quasi-isotropic laminate.

the fibers at the net section, as shown in Fig. 1, but away from the hole edge, at a tensile load of 1.01. Since there are no fibers in another layer to suppress this matrix failure, this matrix failure constitutes failure of the laminate. Indeed, with the matrix cracking, the laminate will disintegrate much like the $(0_8)_S$, namely, with many cracks in the matrix following the fiber direction. Table 1 summarizes these findings, as well as other results to be discussed shortly. The normalized failure loads as predicted by the Tsai-Wu criterion are shown in parenthesis in the table. Pertinent comments regarding failure of each laminate are included.

The next logical step in the curvilinear design is to eliminate the matrix tension failure perpendicular to the fibers in the $(C_8)_S$ laminate. In the $(C_8)_S$, the matrix tension failure perpendicular to the fibers is a result of a Poisson effect in the plate. The high strain in the x direction at the net-section hole edge causes a large contraction strain in the y direction at the hole edge. Away from the hole edge but still at the net section, the strain in the x direction is not as large. As a result, the Poisson contraction in the y direction is not as large. With a large y-direction Poisson contraction at the hole edge and a smaller y-direction Poisson contraction away from the hole edge, the material is actually subjected to a tensile stress in the y direction. This tension is enough to crack the matrix. If a small amount of reinforcement is added perpendicular to the fibers in this region, the matrix tension cracking would be suppressed there. This idea is pursued next.

To keep all laminates 16 layers, so the weights and thicknesses of all laminates studied are identical, 14 layers in the primary load direction are chosen to be curvilinear and 2 are chosen to be orthogonal to these layers. The notation adopted is $(O/C_7)_S$. The O denotes layers orthogonal to the curvilinear layers. Though it is felt that this change in the laminate construction would not significantly alter the principal stress directions relative to the $(C_8)_S$ laminate, the iteration procedure was repeated, starting with the fiber directions for the $(C_8)_S$ design. As expected, after the iteration produced a converged solution, the fiber directions of the $(O/C_7)_S$ and the $(C_8)_S$ were practically identical. However, for the $(O/C_7)_S$, there were two layers that were everywhere orthogonal to the C layers.

The addition of the orthogonal layers more than suppresses the matrix tension cracking. The $(O/C_7)_S$ laminate is predicted to fail at a normalized load of 1.89, the fibers at the net-section hole edge limiting the load. Having fibers limiting the load, as opposed to having matrix limiting the load, is making good use of the concept of fiber reinforcing. Having a fiber-reinforced material fail in a way other than fiber failure would seem to be an inefficient use of the material. In Table 1, the superior load capacity of the orthogonal-curvilinear design contrasts the load capacities of $(0_8)_S$ and $(C_8)_S$ designs.

Though the $(O/C_7)_S$ laminate accomplishes the goal of preventing matrix failure, it suffers from two distinct problems. First, it is very susceptible to a shear failure. Any slight misalignment of the tensile loading, or any amount of shear introduced at the plate boundaries, would most likely crack the matrix. The likelihood of having a pure tensile loading, as has been assumed, is small; therefore, shear is an issue. Second, the manufacturing of a laminate with fibers in an orthogonal grid could be a problem. It is probably safe to say that any manufacturing technique would result in less than perfect placement of the fibers. The fibers could only be oriented in the desired direction to within a specified tolerance. This would result in a slight misalignment of the fibers. Because of this slight misalignment, there could be unwanted tensile or shear stresses in the matrix. This would lead to matrix failure and negate any gains made by using curvilinear fibers. To counter any unwanted shear loading, or any misalignment of the fibers, a pair of ± 45-deg layers are used in the laminate instead of using an orthogonal layer arrangement. The result is a $(\pm 45/C_6)_S$ laminate. Adding the ± 45-deg layers to a laminate would be straightforward from a manufacturing viewpoint. Furthermore, the off-axis layers tend to serve as a sandwich to hold the curvilinear layers together, a desirable feature. With the off-axis layers in the laminate, the directions of the curvilinear layers would not necessarily be the same as the directions of the curvilinear layers with the orthogonal layers. Therefore, the iteration procedure is again used to find the specific fiber directions in each element.

The iteration procedure applied to the $(\pm 45/C_6)_S$ laminate results in a curvilinear fiber format that is not appreciably different from the curvilinear format for the $(C_8)_S$ and $(O/C_7)_S$ designs. The analysis indicates that the $(\pm 45/C_6)_S$ laminate sustains a load level of 1.60. Failure is due to tension in the fiber direction in the curvilinear layers at the net-section hole edge.

Figure 2 illustrates the directions of the curvilinear fibers in the $(\pm 45/C_6)_S$ design. By way of this figure, the finite-element discretization is also shown. Several other points are illustrated by the figure. First, the figure illustrates the assumption in the analysis that the fiber direction within each element is constant. However, even with this restriction, the fiber trajectories

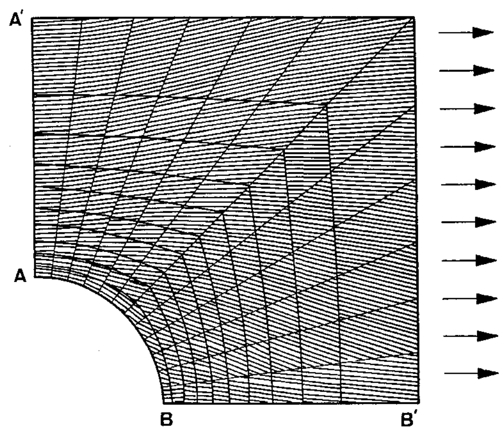

Fig. 2 Directions of curvilinear fibers in $(\pm 45/C_6)_S$ plate.

are smooth. Second, the figure illustrates that the fiber trajectories involve fairly gentle curves. For a structure with a window-sized hole, radical changes in fiber direction with location do not occur. It is therefore conceivable that laminates with the curvilinear format can be manufactured.

The straight-line counterpart to the $(\pm 45/C_6)_S$ design is a $(\pm 45/0_6)_S$ design. Both designs have four off-axis layers at ± 45 deg and 12 load-bearing layers. Application of the maximum strain criterion to this straight-line counterpart indicates that it does not have as much tensile capacity as the curvilinear design. Specifically, the normalized load capacity of the $(\pm 45/0_6)_S$ is 1.27. The failure mode of the straight-line design is fiber failure in the 0-deg layers at the net-section hold edge. The Tsai-Wu criterion predicts some shear interaction in the failure process and, hence, a load less than the maximum strain load. What is puzzling is the following. Near the net-section hold edge, the $(\pm 45/C_6)_S$ and the $(\pm 45/0_6)_S$ designs look identical. In the curvilinear design, the curvilinear fibers pass by the net-section perpendicular to a line from the hole edge to the plate edge, line AA′ in Fig. 2. In the straight-line design, the 0-deg fibers also pass by the net-section perpendicular to that same line. Both designs have ± 45-deg layers at the net-section hold edge. Locally, then, near the net-section hole edge, where failure is predicted to occur in both cases, the two laminates look identical. Yet their load capacities are different!

Two other designs further illustrate this point, and also demonstrate the advantage of the curvilinear design. These two designs are a $(\pm 45/C_2)_{2S}$ and a $(\pm 45/0_2)_{2S}$ laminate. Both designs have the same number of ± 45-deg layers and both have the same number of what might be considered load-bearing layers. Yet the curvilinear design has a normalized load capacity of 1.33, and the straight-line design has a capacity of 1.20. As can be seen in Table 1, both designs fail due to fiber failure, though according to the Tsai-Wu criterion, there is some shear failure in the $(\pm 45/0_2)_{2S}$ laminate. Again, these two laminates have similar fiber orientations near the net-section hold edge, point A, where they both fail, yet the curvilinear design has a higher load capacity.

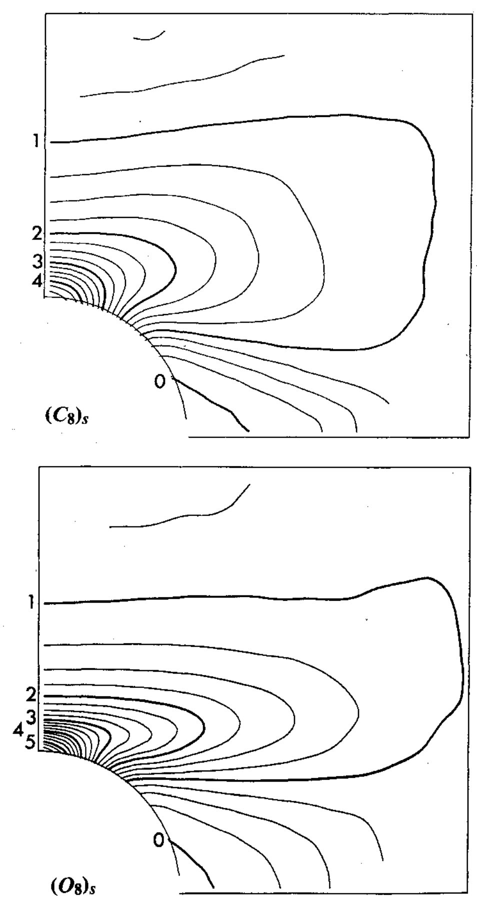

Fig. 3 Contours of normalized stress resultant N_x.

Figure 3 provides insight into this dilemma. This figure illustrates contours of the stress resultant N_x for two different laminates. For exaggeration of the effect and to make a point, the two laminates chosen are the $(0_8)_S$ and the $(C_8)_S$ laminates. The value of N_x applied at the ends of the plate is the same in each case, the the contours in the plate have been normalized by this load. Hence, these contours illustrate a stress concentration effect. Two characteristics are clear from the figure. First, the stress concentration is lower for the $(C_8)_S$ laminate than the $(0_8)_S$ laminate. Second, the distribution of the contours is different. The contours for the $(C_8)_S$ laminate are not as concentrated at the net-section hold edge as they are for the $(0_8)_S$ laminate. Additionally, the shapes of the $(C_8)_S$ contours are different than the shapes of the $(0_8)_S$ contours. Specifically, the contour loops of the $(C_8)_S$ design are broader and less narrow than the contour loops of the $(0_8)_S$ design. This indicates that the curvilinear fibers of the $(C_8)_S$ design tend to move the load outward around the hole more than the straight-line fibers of the $(0_8)_S$ design do. With this influence, the material away from the hole is being used more effectively in the $(C_8)_S$ than with the $(0_8)_S$ laminate. This effect translates to the other curvilinear designs.

It should be mentioned that recently Katz et al.[8] used sequential linear programming in an optimization scheme to investigate performance increases using varying fiber direction in a plate with a hole. The geometry and material properties used by Katz et al.[8] were identical to those considered here. A maximum strain criterion was also used as a measure of performance. The fiber orientation was allowed to vary in seven regions of the plate. Fiber orientations in the seven regions were varied in a systematic way by the optimization scheme. Predicted performance increases and predicted fiber directions compared quite well with the results discussed here.

Buckling Considerations

With a design for improved tensile performance using the curvilinear established, the influence of these designs on buckling resistance is investigated. The approach here is to simply compute the buckling loads of the plates. It is assumed that the plates are simply supported on all four edges. Table 1 summarizes the buckling load results for the plates. Again, the quasi-isotropic laminate with identical geometry is used as a normalization datum.

Several interesting results are found when considering the buckling strength. First, the $(0/C_7)_s$ laminate, which is so much better than the quasi-isotropic laminate in tension, has a buckling load about 45% less than the buckling load of a quasi-isotropic laminate. Second, none of the laminates studied have as high a buckling load as the quasi-isotropic laminate. The $(\pm 45/C_2)_{2S}$ and $(\pm 45/0_2)_{2S}$ laminates have buckling loads quite close to the buckling load for a quasi-isotropic laminate. This is presumable due to these two laminates having the same number of (± 45) layers as the quasi-isotropic laminate. A third point to be observed is that although the buckling loads of the $(\pm 45/C_6)_S$ and $(\pm 45/0_6)_S$ laminates are lower than the buckling load of the quasi-isotropic laminate, the laminate with the curvilinear fiber format has a slightly higher buckling load. If this information is combined with the fact that, in tension the $(\pm 45/C_6)_S$ is better than the $(\pm 45/0_6)_S$, and clearly better than the quasi-isotropic laminate, then the $(\pm 45/C_6)_S$ is a laminate that deserves consideration. The gains in using a $(\pm 45/C_2)_{2S}$ rather than a $(\pm 45/0_2)_{2S}$ are not as significant.

Though the results presented are for a specific geometry, as stated earlier, the results are similar for other geometries. References 9 and 10 document these findings.

Additional Comments and Conclusions

Presented has been an idea for possibly improving structural efficiency using fiber-reinforced materials. Specifically, the notion of using a curvilinear fiber format in flat plates with central circular holes has been studied. The paper is not all encompassing, and it was not intended to be that way. This note simply suggests that the curvilinear format has the potential for using reinforcing fibers more effectively. Designs using the curvilinear format have been contrasted with conventional straight-line design counterparts. Results have been compared with a standard quasi-isotropic design. Tensile and compressive buckling loads have been studied. It can be concluded that in tension the curvilinear designs studied lead to improved performance. In compression, the buckling loads are not as high as they are for quasi-isotropic laminates, but the buckling loads for the curvilinear design are no lower than the buckling loads for their straight-line non-quasi-isotropic counterparts.

Before closing, a comment should be made regarding manufacturing. As with many designs, the curvilinear configurations discussed here may present some manufacturing problems. For example, the $(\pm 45/C_6)_S$ design calls for the fiber angles along line BB′ to not be horizontal. When considering a complete plate, rather than just one quarter, this is a problem. For a complete plate, the fibers along line BB′ must be horizontal. Along line BB′ there cannot be a slightly negative angle specified by the top quarter of the plate and a slightly positive angle specified by the bottom quarter. In practice, the fiber angles in the elements along line BB′ must be adjusted to make them horizontal. This may not impact the increased efficiency. Sensitivity analyses, to determine which areas of the plate are least effected by deviations of the fiber angle from the ideal, can be conducted. It is suspected that the load capacity is not particularly sensitive to fiber angle along line BB′. Thus, the fiber angles can be adjusted to achieve manufacturing compatibility and not significantly effect performance.

Also regarding manufacturing, Knoblach[11] recently investigated the use of electric fields to align short chopped fibers in specific directions while the material was being manufactured. There were problems with the electrodes and fiber-fiber interaction but the concept worked. Plates were produced that had better stiffness and strength properties than plates with random fiber orientation, i.e., quasi-isotropic.

More work needs to be done. Studies are needed to pursue laminates with other combinations of straight and curvilinear layers. Currently, studies are underway to investigate the use of the curvilinear format to increase the buckling resistance. Manufacturing issues are also being addressed. The character of postbuckling performance and failure with he curvilinear format also deserves attention. These developments will be reported on at a later date as work progresses.

Acknowledgments

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Analytical Model Improvement Using Measured Modes and Submatrices

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Introduction

THERE are numerous methods available in the literature describing how to improve an analytical model of a linear structure in order to match the experimentally identified mode shapes and frequencies. These methods are characterized by the assumptions and goals made with respect to three sets of data: an analytical mass matrix, an analytical stiffness matrix, and a set of measured modes. Assuming that the analytical mass matrix is correct, methods for adjusting the measured modes to achieve orthogonality have been proposed e.g., Ref. 1. The opposite approach, which has been also taken, assumes that the measured modes are correct and adjusts the analytical mass matrix.[2] When the orthogonality condition between the analytical mass matrix and the measured modes is satisfied, various methods can be employed to correct the analytical stiffness matrix.[1-4]

This Note addresses the procedure of correcting the analytical mass and stiffness matrices simultaneously to match the measured modes instead of updating the mass matrix first and the stiffness matrix subsequently. In order to achieve this, the previously proposed submatrix approach to the analytical stiffness matrix adjustment[5] is extended while preserving the characteristics of the method. Thus, the connectivity and the consistency of the stiffness matrix are preserved and the mass orthogonalized mode shapes are not required. A computationally efficient pseudoinverse solution is employed to solve the resulting system of linear equations for the submatrix scaling factors that are used to correct the mass and stiffness matrices. The method also incorporates a capability of reducing finite

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